ON THE COST-EFFECTIVENESS OF MULTISTAGE DEPLOYMENT OF WIDE AREA MONITORING SYSTEMS IN WEAK NETWORKS UNDER LIMITED CHANNEL AVAILABILITY

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ABSTRACT

Utilities operating in weak networks can benefit from Wide Area Monitoring Systems (WAMS) installations to improve grid voltage reliability through increased network observability. Multistage placements with long-term full observability seem like a fiscally viable alternative, but are likely to cost more in the end. Here, we focus on the cost-effectiveness of using WAMS in developing countries, especially in terms of fiscal capability. We introduced some limited-budget, limited-channel WAMS placement problems, and discuss some factors affecting continued installations. In the end, we concluded that weak networks can also enjoy the benefits but must be aware of certain practical factors in multistage deployment when budget is of concern.

INTRODUCTION

Wide Area Monitoring Systems (WAMS) are becoming increasing ubiquitous on account of the many benefits they offer. These benefits include, amongst others, increased awareness of real-time events in the grid, fault detection and classification, model validation, and better accuracy of state estimation. On the distribution network, micro-Phasor Measurements Units (μPMUs), a form of WAMS may be used in this regard with the benefit at this voltage level discussed in [1]. In general, all forms of PMUs measure time-synchronised and high-resolution voltage and current phasors which are fundamental practical quantities for general grid health monitoring and control. For monitoring purposes, WAMS are placed on specific buses in the grid, for full or partial observability depending on the objective and structure of the electricity market. The concept of full and partial observability in this sense being to be able to obtain a voltage and current phasor measurement of all or selected network buses and branches respectively.

In many developed countries, WAMS are majorly installed on the transmission networks, and to a lesser extent, on the distribution networks. This is because distribution network operators (DNOs) are highly reluctant to install expensive WAMS on their networks and would often require a good justification of the benefits they offer to justify the costs of investments. In addition, grid visibility is often enhanced with an observability of the wide area at higher voltage levels. There is little or no investment in WAMS in many with attendant consequences of the loss of the higher measurement resolution and visibility they offer. Many failures in the upstream transmission systems have led to high unreliability in bulk power supply to the distribution grid. In consequence, the distribution companies (DISCOs) have either had to fail to supply power to residential and industrial customers, or resorted to load scheduling among different distribution areas. Furthermore, electricity theft caused by low penetration of metering or monitoring devices, as well as frequent electrical component and line failures are rampant in the lower distribution grid. It then follows from this reasoning that all grid operators would benefit from higher grid visibility in order to improve the reliability of their services.

The main goal of the liberated and not-so-liberated market is to make profit. Therefore, any intended investments in any infrastructure must indeed justify its benefits in that market, regardless of its sophistication. The costs associated with PMU placements include the hardware unit purchase price, communication costs, design and engineering costs, costs of labour and materials, costs of ancillary devices such as global positioning systems (GPS) and networking devices, costs of disrupting a substation, associated costs of installing phasor data concentrators (PDCs) [2]. The PDCs serve as data storage and archiving functions for offline analysis and backup in the event of PMU failure. On account of the high costs involved in WAMS installations, the number of monitoring devices installed in a network is upper-bounded by the benefits associated with these installations, such that at a certain number $n > N_{PMU}$ PMUs, any further installation offers no additional benefits [3].

In varying degrees, PMU Placement (PP) algorithms aim to minimise total placement costs with the condition that all buses must be observable. Observability in this sense does not imply that WAMS are installed on all buses. Indeed, all buses connected to a bus b_i, with a PMU m_i installed are deemed observable because their measurements can be indirectly estimated from the voltage drop obtained from their known line impedances. Most literatures are primarily focused on minimising the total cost of placement, without a realistic consideration of what the final cost might be, in terms of affordability and necessity. The goal being to monitor all buses in a one-time placement (see e.g. [1]), or over time spreading out the number of placement per time using incremental placement over a period of time, as obtained in multistage scheduling methods of [4]–[6] with a long-term focus on full observability. However, financial realities compel probing of the full observability realities. For this reason, some papers have addressed the placing of the PMUs only on certain buses based on the criticality or relative importance of these buses.

The multistage algorithms [4]–[6] have examined the placement problem in different ways. In [4],[5] the focus has been on quantifying the reliability of electrical line and communication, and this has been used as a factor for determining candidate PMU buses. Some of the earliest work on multistage placement was done in [6], where the number of components placed at each stage is pre-determined such that the total number of placements is equal to the total determined from a system-wide optimization with full-observability objective. To the best of our knowledge, many multistage algorithms have not examined the possibility of the decreasing unreliability of already-installed equipment, the consequences of a limited budget and the loss of inflationary rates on budgets, nor have the possibility of adopting WAMS in weak networks, The contribution of this paper are, using multistage algorithms, to examine the possibilities of the adoption of WAMS in budget-constrained market and to examine the factors influencing incremental placement of PMUs such as equipment degradation and time lapse between placements. PMU channel limits also affect the number of current measurements by a PMU, and we model this effect by formulating a budget-constrained channel-constrained multistage placement method using integer linear programming.

BACKGROUND

The integer linear programing Optimal PMU (ILP-OPP) placement problem determines the optimal but non-unique set of buses where a PMU may be placed such that all network buses are fully observable. The ILP approach guaranteeing optimality but non-uniqueness of solutions, meaning that other placement on other sets of buses different from the ones obtained in the solution may result in full observability, with the same number of PMUs.

$$\min_{x_j} \sum_{j=1}^{n} c_j x_j$$

$$s.t \begin{cases} \sum_{j=1}^{n} A_{ij} x_j \geq R + 1, & \forall i = 1,2,\ldots,n \\ x_j \in \{0,1\} \end{cases}$$

$$A_{ij} = \begin{cases} 1, & if\ i = j \\ 1, & if\ i\ is\ connected\ to\ j \\ 0, & otherwise \end{cases} \quad (1)$$

x_j is a binary variable whose value is 1 or 0 depending on whether or not, respectively, a PMU is placed on a bus i. Some form of reliability of observability may be incorporated with the number of redundancy R on the RHS of the constraint. In the case of no redundancy ($R = 0$), the optimal PMU placement (OPP) usually results in a relatively fewer number of PMU placements, but still achieves full observability. With $R \geq 1$, the OPP results in a higher number of placements, full observability, and increased guarantee of continued observability with the loss or failure of a measurement unit. For limited budget consideration, it is necessary to add a cost constraint to the optimisation setup above. Realistically, the costs,

$$c_T = \sum_{j=1}^{n_{placed}} c_j \leq B \quad (2)$$

$$c_j = c_{procurement}^j + c_{comission}^j + c_{install}^j$$

Where c_T, $c_{procurement}^j$, $c_{comission}^j$, $c_{install}^j$ are the total cost and the costs of procurement, commissioning, and installing of a single PMU at a bus j. It should be noted that the dual constraints of full observability and limited budget might result in an infeasible optimisation problem, depending on the size of the budget and the budget consideration. Traditionally, this constraint is neglected with the assumption that funds are available to cover the full costs of placements, or that the quest for full observability should be delayed until such a time as the costs for full observability may be fully covered.

THE MULTISTAGE PLACEMENT PROBLEM

Multistage Placement with Limited Budget Consideration – Base Case

The placement problem with budgetary constraints is focused on determining the number of PMUs which may be realistically placed considering unavoidable financial constraints. First, we follow many traditional methods of placing with the imposition of full observability on our optimisation. We follow from the ideas in [6] and [7], and minimise the total cost of placement to determine the total number of PMUs needed for full observability.

Step1: OPP-Budget-Base

$$N_{PMU}^{fullobsv} = \min_{x_j} \sum_{j=1}^{n} c_j x_j$$

$$s.t \begin{cases} \sum_{j=1}^{n} A_{ij} x_j + \sum_{j=1}^{n} A_{ij} z_j y_{ij} \geq R + 1, & \forall i = 1,2,\ldots,n \\ \sum_{i=1}^{n} A_{ij} y_{ij} = z_j, & \forall j = 1,2,\ldots,n \\ x_j, y_{ij} \in \{0,1\} \end{cases} \quad (3)$$

The auxiliary binary variable y_{ij} is introduced in [7] to model the ZI buses. z_j is a vector whose element is 1 if bus j is a zero-injection bus, and which is 0 otherwise. From the results of the optimisation in (3) we obtain a binary solution to the placement.

The second step is to proceed to the core of the multistage budget-constrained placement problem. In this step, we seek a solution such that the number of PMUs placed at every stage is within a certain financial limit but maximises the observability at that stage.

Step2:

$$\max \sum_{j=1}^{n} u_j$$

$$s.t \begin{cases} \sum_{j=1}^{n} A_{ij}x_j + \sum_{j=1}^{n} A_{ij}z_j y_{ij} \geq u_j, \ \forall i = 1,2,\dots,n \\ \sum_{i=1}^{n} A_{ij}y_{ij} = z_j, \quad \forall j = 1,2,\dots,n \\ \sum_{i=1}^{n} M_{ij}x_j = \mathbf{0}, \quad \forall j = 1,2,\dots,n \\ \sum_{i=1}^{n}\sum_{j=1}^{n} \breve{M}_{ij}c_j x_j \leq B^k_{phase} \\ x^*{}_j, u_j, y_{ij,} \in \{0,1\} \end{cases} \quad (4)$$

Where $u_j \in \{0,1\}$ is the binary variable representing the observability state of bus j. The third constraint above ensures that only buses present in the original solutions are candidates in the intermediary stage, since PMUs cannot be expected to be uninstalled after installation at intermediate stages. In addition, this also ensures that the total number of PMUs across all stages is equal to $N^{fullobsv}_{PMU}$ at the end of all placements. The parameter matrix, $M_{ij} = 0$ if $x_j = 0$ in (2), and 1 otherwise. On the other hand, $\breve{M}_{ij} = 1$ if $x_j = 1$ in (2), and 0 otherwise.

Multistage Placement with Limited Budget and Limited Channels Considerations

Most algorithms rely on the assumption that the PMU placed on a bus b_i can measure the current along all branches connected to it. In reality, this is not the case. Any result obtained from placements that do not account for channel limits are bound to be inapplicable in practice. For instance, a PMU with two channels ($L = 2$) can measure at most currents along L number of branches connected to its bus. Therefore, to account for PMU channel limit, we use the expression from [8]:

Step 1: OPP-Multichannel

$$x^* = \min_{x_{jr}} \sum_{j=1}^{n}\sum_{r=1}^{r_j^L} c_L x_{jr}$$

$$s.t \begin{cases} \sum_{j=1}^{n}\sum_{r=1}^{r_j^L} b_{ijr}x_{jr} \sum_{j=1}^{n} A_{ij}z_j y_{ij} = R+1, \ \forall i = 1,2,\dots,n \\ \sum_{i=1}^{n} A_{ij}y_{ij} \geq z_j, \quad \forall j = 1,2,\dots,n \\ x_j, y_{ij} \in \{0,1\} \end{cases} \quad (5)$$

b_{ijr} is a three-dimensional matrix representing the subsets of all possible branch subsets S_{jr} with a centre at bus b_i which may be used to measure the current flowing through the line connecting bus i to bus j. r_j^L is the total of all possible combinations of branches whose subsets elements is usually less than or equal to L. A more comprehensive explanation of this setup can be obtained in [8].

Step 2: OPP-Budget-Multichannel

Next, we introduce a multistage limited budget, limited channel placement, which maximises the observability of bus and branches given that the PMU connected to a bus i may not be able to observe the current among all its branches connected to it.

$$\max \sum_{j=1}^{n}\sum_{r=1}^{r_j^l} u_{jr}$$

$$s.t \begin{cases} \sum_{j=1}^{n}\sum_{r=1}^{r_j^L} b_{ijr}x_{jr} \sum_{j=1}^{n} A_{ij}z_j y_{ij} \geq u_{jr}, \ \forall i = 1,2,\dots,n \\ \sum_{i=1}^{n} A_{ij}y_{ij} = z_j, \quad \forall j = 1,2,\dots,n \\ \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{r=1}^{r_j^L} M_{ijr}x_{jr} = 0 \\ \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{r=1}^{r_j^L} \breve{M}_{ijr}c_{jr}x_{jr} \leq B^k_{phase} \\ x_{jr}, y_{ij}, u_{jr} \in \{0,1\} \end{cases} \quad (6)$$

In (5) and (6) above are the homogenous limited channel forms of (3) and (4) and the variables u_{jr}, x_{jr}, M_{ijr}, $\breve{M}_{ijr}$, and c_{jr} are defined in similar manners. In particular $M_{ijr} = 1$ when $x_{jr} = 0$ in (5) and 1 otherwise, while $\breve{M}_{ijr} = 0$ when $x_{jr} = 1$ in (5) and 1 otherwise.

Factors affecting assigned budgets at each placement stage

A viable installation planning is influenced by a practical knowledge of the total cost of procuring, installing, and commissioning a unit PMU, with all expectations of further increases in costs due to the peculiarities of particular networks taken into consideration. The total amount of investment in WAMS in a given time period can be expected to be limited by the maximum allowable amount which an operator may be willing to invest in a given time period such that the cost of investment does not exceed the benefits offered by increased grid visibility in the long term [9]. The budget per phase is more highly influenced by short term planning and may be assumed to be evenly spread throughout a future time period T years. Suppose, in the first stage, that $N^{fullobsv}_{PMU}$ PMUs are required for full observability, and a realistic cost c_T ($) is estimated for a working installation, then the budget at each stage,

$$B^k_{phase}(\$) = \frac{c_T \times N^{fullobsv}_{PMU}}{T} \quad (7)$$

From the optimisation setups above, it is clear that the number of installed PMUs in the multistage placements are determined by the budget B^k_{phase} allocated to that phase as well the unit installation cost. The number of installations over all k stages cumulatively determine the time T for the realisation of full grid observability. If at any kth stage, $B^k_{phase} < c_T$, then no installation is done at that time, the budget can be carried over and $B^{k+1}_{phase^{new}} = B^{k+1}_{phase} + B^k_{phase}$. Following from the foregoing and depending on the simple metric in (7) as well as other extenuating circumstances, T may be used as an indicator in a WAMS installation model. This would include the operator's reluctance to further invest in WAMS due to high WAMS costs/benefit ratio in spite of initially committing to it, high substation shutdown costs, or a lack of expertise in WAMS installations.

Deteriorating Performance of Already Installed Components

A good incremental or multistage PMU placement algorithm models for limited budget consideration must account for time lapse between successive placements. Thus, the probability that a PMU installed at a location x_i will continue to work for a number of years until the PMU determined by an incremental placement is installed at another location x_{i+1} is not fully guaranteed. Although the reliability of measurement devices is typically high, budgetary allocations determine if the unreliability of already installed PMUs would not have deteriorated to such a level as to warrant significant maintenance costs. To allow for this, a certain percentage of the total budget for the current stage is allocated to maintaining existing components.

Full Regional Observability or Maximisation of Observability

If full observability is not desired, some elements of the vector $R+1$ may be set to 0 at the desired buses in the OPP. For the maximisation of observability with long term full observability objective, at each stage of Step 2 of the BCM-PP, the optimisation seeks to maximise the observability across the network. When the intention is to maximise observability across a particular region of the network, it is necessary to modify $M_{ij(r)}$ in (4),(6) such that only candidate solutions in the desired regions are nonzero.

Inflation Rate and Improvement in Technology

Year on year inflation may affect the projected cost of the PMU at any future $(k+1)th$ stage. This financial reality has been overlooked in current literature on multistage placements. Inflation generally reduces the purchasing power of a currency, affects general costs of installations, and results in a decrease in anticipated installations. One may account for its effect by adjusting for it early on in the planning stage by varying the cost over each incremental phase using a knowledge of past inflation rates, or by varying the cost by the inflation rate over each incremental phase.

On the other hand, an improvement in PMU manufacturing technology may result in the reduction of unit procurement costs for future procurements, or results in advanced communication and measurement protocols with attendant interoperability issues with already installed devices.

Algorithm 1: Budget-Constrained Multistage Placement Algorithm (BCM-PP)

Max Number of PMUs
0.1 Obtain System Topology, number of Redundancy, $\boldsymbol{R}$ and ZI buses, $\boldsymbol{z_j}$.
0.2 Solve the OPP-Base or OPP-Multichannel, minimising the cost of installing PMUs for grid monitoring, but focusing on full observability at this point.
0.3 Determine the number of PMU required for full observability, $\boldsymbol{N_{PMU}^{fullobsv}}$ and candidate locations, $\boldsymbol{x^*}$ for their installations from the solution of the above.
***Stage k* = 1: Max Observability, Factored Costs**
1.1 Obtain budget for the stage, $\boldsymbol{B_{phase}^{k}}$, cost price per PMU, $\boldsymbol{C_l^{p1}}$, and cost of installation per PMU, $\boldsymbol{C_l^{inst1}}$, as well as $\boldsymbol{R}$, $\boldsymbol{z_j}$ from the previous steps. Calculate $\boldsymbol{C^1 = C_l^{p1} + C_l^{inst1}}$
1.2 Solve the PP-Budget-Base or PP-Budget-Multichannel, maximising the number of buses which can be monitored in a certain region at Stage $\boldsymbol{k-1}$.
1.3 Obtain the number of PMUs required for stage 1, $\boldsymbol{N_{PMU}^{STGk}}$
1.4 Obtain the reliability, $\boldsymbol{{r_k}^i}$ for each PMU placed in stage 1, $\boldsymbol{x^{STGk}}$ and their respective unreliability, $\boldsymbol{{q_i}^1 = 1 - {r_i}^1}$.
1.5 Obtain the number of PMUs, $\boldsymbol{N_{PMU}^{STG1-R} = N_{PMU}^{fullobsv} - N_{PMU}^{STGk}}$ left to install.
1.6 Obtain $\boldsymbol{B_{bal1} = B_{phase1} - B_{cost1}}$
1.7 $\boldsymbol{B_{phase}^{2} = B_{phase}^{1} + B_{bal}^{1}}$
***Stage k* ≥ 2: : Max Observability, Factored Costs**
2.1 Reduce $\boldsymbol{r_i^1, \ldots, r_i^{k-1}}$, by $\boldsymbol{y}$% to model the increase in the unreliability of the installed components from Stage **1** - Stage $\boldsymbol{k-1}$
2.2 Repeat Steps $\boldsymbol{1.1 - 1.4}$ of the previous of Stage $\boldsymbol{k-1}$
2.3 Obtain $\boldsymbol{\chi^k = (1-h)}$% as the inflation factor.
2.4 Calculate the maintenance factor $\boldsymbol{\psi^k = q^1 \times \sum_{l=1}^{N_{PMU}^{STG1}} C_l^1 {x_l}^{STG1} + \cdots + q^{k-1} \sum_{l=1}^{N_{PMU}^{STGk-1}} C_l^{k-1} {x_l}^{STGk-1}}$
2.5 Calculate the savings due to improvement in Technology, $\boldsymbol{\eta^k = Z\% \times \left(\sum_{l=1}^{N_{PMU}^{STGk}} C_l^k {x_l}^{STGk}\right)}$
2.6 Calculate the budget for step $\boldsymbol{k+1}$ as $\boldsymbol{B_{bal}^k = B_{phase}^k - B_{cost}^k}$.
2.7 $\boldsymbol{B_{phasek+1} = \chi^k (B_{phase}^{k+1} + B_{balk} + \eta^k - \psi^k)}$
2.8 Obtain the number of PMUs, $\boldsymbol{N_{PMU}^{STGk-R} = N_{PMU}^{fullobsv} - \sum_{i=1}^{k} N_{PMU}^{STGk}}$ left to install.
2.9 If $\boldsymbol{N_{PMU}^{STGk-R} = 0}$, **stop,** else $\boldsymbol{k = k+1}$

CASE STUDY

Table 1 shows a hypothetical quantification of the factors which may influence BCM-PP. The reliability of installed electrical components is expected to decrease with the maintenance factor changing in the opposite direction. Conversely, the chances that improvement due to technology would reduce procurement expenses increases.

Table 1: Hypothetical values of some practical factors influencing Stage budgetary provisions

T	Reliability	Inflation rate	Maintenance factor	Technology Improvement
1	0.99997	1%	0.00003	0.005%
2	0.99987	1%	0.00013	1%
3	0.99970	1%	0.0003	1%

A simple IEEE 14-bus network is used to illustrate the above algorithm. The network has a zero-injection bus at Bus 7. In Stage 1, the solution of the OPP-Base or OPP-Multichannel placed the buses on $2, 6, 9$ with the consideration of zero-injection buses as in [6]. At an average cost of $\$40{,}000$ for procurement, installation and commissioning, a total of $\$120{,}000$ would be required for a one-time installation. If this is available, it is not necessary to apply the budget-constrained algorithm of Stage 2. In large networks as well as in weak ones, with many candidate solutions, budgets are almost often constrained.

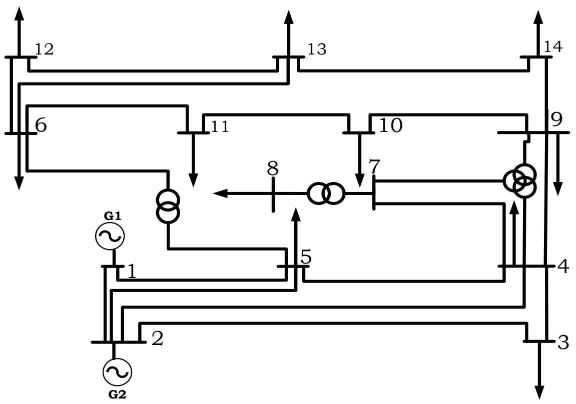

Fig 1 IEEE 14-Bus Test System

With a uniform total cost of PMU installation $\$50{,}000$ across candidate buses $2, 6, 9$, and a budget of $\$63{,}000$/stage the BCPP places Stage 1 PMU at Bus 2, similar to the result obtained in [6]. As can be expected, at a lower budget of $\$47{,}000$, the BCPP has no solution. Results become significantly different from [6] at a budget of $\$100{,}000$, where the candidate solutions are placed at $\{2,6\}$. At Stage 2 with $\boldsymbol{B_{phase1}} = \$63{,}000 +$/stage, the BCM-PP places the next PMU at bus 9. Clearly, at the last stage, bus 6 is the only other choice. In practice, the time lapse between each placement is not a given and may be difficult to model.

At any stage $k+1$, solutions obtained in the previous stage k may be removed from the candidate solution by setting the diagonal $M_{ii} = 1$ in (4) and then solving the BCM-PP. This process can also be used to remove solutions from the equation to suit operator bias, or topological changes at any stage. As the solution to the OPP gives a non-unique but optimal solution, it is possible to obtain other candidate buses, which may not be part of the original solutions, but still have full observability at the end of all planned placements. If this is not the case, the solution at that bus may be deferred, but not completely abandoned.

Depending on realistic values of factors outlined in Table 1, the budgetary provisions at each stage may increase or decrease even with a provision of a uniform budgetary allocation according to (6). For instance, if inflation rate is low and the reliability of the system is consistently high, the budget for the following years may, at worst, remain the same. If the reverse is the case, some budgetary allocations for successive stages may be spent on maintaining the previously installed ones, and consequently, the overall cost of placement would increase significantly.

CONCLUSION

We have introduced a budget-constrained multistage placement problem (BCM-PP) which is aimed primarily at encouraging the deployment of WAMS in weak networks We pointed out some practical factors affecting multistage programming and formulated a simple equation which may be used to obtain a realistic expectation of the actual stage financial limit at each stage. Expectations may be different from realities if these factors are not considered. The effect of line outages and the availability of communication infrastructures in substations at each stage are not considered although their effect may be easily incorporated into the relationship we have introduced.

We conclude that the determination of the candidate solution at any stage is a function of all costs associated with the PMU. With good planning, weak networks can also enjoy the benefits of WAMS.

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