# AGENT BASED DISTRIBUTED OPTIMAL POWER FLOW USING ADMM METHOD

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## ABSTRACT

*In this paper, we address the AC optimal power flow (OPF) problem of a microgrid with a distributed mathematical strategy. The goal is to find an optimal operating point of a grid with limited communications. A multi-agent system was designed to implement a state-of-the-art distributed optimization method, which is alternating direction method of multiplier (ADMM). An agent, hosted in a processor installed at grid nodes, sends optimal set points to connected controllable devices. In this distributed strategy, agents need only local and neighbourhood data but can give global solutions. A 6 bus system is used to validate the agent based distributed algorithm with 6 agents at locations of nodes.*

## INTRODUCTION

Conventionally, centralized OPF problems had been focusing on the transmission network, which is responsible of transfer power from centralized power plant to large loads and cities. However, the power grid is undergoing an profound mutation. The modern smart grid, especially at the distribution level and within microgrids, is a network hosting distributed energy resources (DERs). The increasing amount of loads and DERs could also lead to the mutation of network topology. Therefore, new challenges in managing these DERs and optimizing their operation arise. Distributed control and optimization are mandatory fields for the next generation of power systems. Distributed rules enhance the scalability, stability and security of grids. Moreover, distributed algorithms also have the potential of keeping the privacy of sensitive information of loads (household, industrial and commercial loads…) or DERs (of different companies).

Recently, many algorithms were developed for distributed control and optimal power flow in power system [1][2]. There are two main approaches of solving OPF in distributed strategy. The first approach is based on augmented Lagrange decomposition, including Dual Decomposition [3], the Alternating Direction Method of Multipliers with Proximal Message Passing [3], Analytical Target Cascading [4], the Auxiliary Problem Principle [5]. The second approach is based on decentralized solution of the Karush-Kuhn-Tucker (KKT) including Optimality Condition Decomposition [6] and Consensus+Innovation [7]. The ADMM used in this paper is proven to be a powerful distributed optimization method [3].

In this paper, we firstly mathematically formulate the OPF problem and decompose it into sub-systems with objective of minimizing power loss. Agents are then designed with server-client structure to implement the ADMM algorithm. Finally, a set of processes representing the multi-agent system is run to optimize the operation of a test case grid with real communication.

## FOMULATION OF DISTRIBUTED OPF PROBLEM

### ADMM Algorithm

Many approaches are based on the ADMM algorithm or its variants. This section provides an overview, see [3] for more details.

We consider a problem which is separated into $N$ subsystems. General form of consensus problem is

$$\min \sum_{i=1}^{n} f_i(x_i)$$
$$\text{subject to } x_i - z_i = 0, \quad i = 1,\dots,N.$$

with local variables $x_1,..,x_N$ and global variable $z$. Each of local variables consists of a number of the components of the global variable $z \in \mathbf{R}^n$ , means that each component of each local variable corresponds to some global variable components. The local variables and global variable achieve consensus when

$$\left(x_i\right)_j = z_{G(i,j)}, \quad i = 1,\dots N, \quad j = 1,\dots,n_i$$

The ADMM algorithm is based on the augmented Lagrange for

$$L_\rho := \sum_{i=1}^{N} f_i(x_i) + y_i^T(x_i - z_i) + \frac{\rho}{2}\|x_i - z_i\|_2^2$$

where $\rho > 0$ is a penalty parameter, $\|\bullet$ is Euclidean norm and $y_i$ is the dual variable.

The ADMM algorithm iteratively minimizes the augmented Lagrange by performing the following updates:

$$x_i^{k+1} := \arg\min_{x_i}\left( f_i(x_i) + y_i^{kT} x_i + \frac{\rho}{2}\|x_i - z_i\|_2^2 \right)$$

$$z_g^{k+1} := \frac{1}{k_g} \sum_{G(i,j)=g} \left( \left(x_i^{k+1}\right)_j + \frac{1}{\rho}\left(y_i^k\right)_j \right)$$

$$y_i^{k+1} := y_i^k + \rho(x_i^{k+1} - z_i^{k+1})$$

where superscripts are the iteration index, $k_g$ is the number of local variable entries that correspond to global variable entry $z_g$.

The $x$ and $y$ updates can be performed independently using only local information. The $z$ update with information from neighbours is local averaging rather than global averaging.

## Formulation of the optimal power flow problem

The OPF problem is solved to find a feasible control, subject to physics and other limitations. The variables are node voltages.

The OPF problem with objective is minimizing active power loss is presented as following:

$$\underset{\hat{V}}{\text{minimize}} \quad \hat{V}^T \cdot \mathbf{Z_P} \cdot \hat{V}$$

Subject to

(at $k = 1, \ldots, N$)

$$P_{\min}^k \le \hat{V}^T \cdot \mathbf{Z_P^k} \cdot \hat{V} \le P_{\max}^k$$

$$Q_{\min}^k \le \hat{V}^T \cdot \mathbf{Z_Q^k} \cdot \hat{V} \le Q_{\max}^k$$

$$V_{\min}^2 \le (V_k^{\mathrm{Re}})^2 + (V_k^{\mathrm{Im}})^2 \le V_{\max}^2$$

where vector $\hat{V}$ indicated by $\hat{V} = \begin{bmatrix} V^{\mathrm{Re}} \\ V^{\mathrm{Im}} \end{bmatrix}$, $V = V^{\mathrm{Re}} + jV^{\mathrm{Im}}$ is the node voltage vector.

Matrices $\mathbf{Z_P}$ and $\mathbf{Z_Q}$ are obtained from:

$$\mathbf{Z_P} = \begin{bmatrix} \mathbf{G} & \mathbf{-B} \\ \mathbf{B} & \mathbf{G} \end{bmatrix}$$

$$\mathbf{Z_Q} = \begin{bmatrix} \mathbf{-B} & \mathbf{-G} \\ \mathbf{G} & \mathbf{-B} \end{bmatrix}$$

where $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$ is the admittance matrix of the network system. $\mathbf{Z_P^k}$ and $\mathbf{Z_Q^k}$ are same size as $\mathbf{Z_P}$ and $\mathbf{Z_Q}$ respectively, obtained by rewrite constraints in the quadratic form.

If node $k$ is the load node:

$$P_{\min}^k = P_{\max}^k = P_{load}^k$$

$$Q_{\min}^k = Q_{\max}^k = Q_{load}^k$$

If node $k$ is the generator node, $P_{\min}^k$, $P_{\max}^k$, $Q_{\min}^k$ and $Q_{\max}^k$ are the active and reactive power limit.

## Distributed formulation of OPF problem

The objective function could be equivalently reformulated as:

$$\hat{V}^T \cdot \mathbf{Z_P} \cdot \hat{V} = \frac{1}{2}\sum_{k=1}^{N} \hat{V}^{kT} \cdot \mathbf{Z_P^k} \cdot \hat{V}^k$$

Therefore we can simply decompose the general OPF problem into N subsystems corresponding to N nodes of network system.

The problem of $kth$ subsystem is defined as:

$$\underset{\hat{V}^k}{\text{minimize}} \quad \frac{1}{2}\hat{V}^{kT} \cdot \mathbf{Z_P^k} \cdot \hat{V}^k$$

Subject to

$$P_{\min}^k \le \hat{V}^T \cdot \mathbf{Z_P^k} \cdot \hat{V} \le P_{\max}^k$$

$$Q_{\min}^k \le \hat{V}^T \cdot \mathbf{Z_Q^k} \cdot \hat{V} \le Q_{\max}^k$$

$$V_{\min}^2 \le (V_k^{\mathrm{Re}})^2 + (V_k^{\mathrm{Im}})^2 \le V_{\max}^2$$

# THE MULTI-AGENT SYSTEM FOR DISTRIBUTED OPF PROBLEM

## Formulation sub-problem for agents

In previous section, the OPF was formulated in the form of general consensus problem. We use the multi-agent system to deploy the ADMM method. Each agent is located at a bus of grid network and takes responsibility of a sub-problem which is dispersed from the decomposing of the global OPF. Agents put at controllable devices will send the set points to device controllers as results of its own problem. In our case, the active power and reactive power are command signals sent from these agents to controllers of generators. We assume that an agent can access local information of its corresponding bus. These local parameters are voltage limit, maximum and minimum power with generator buses or load power with load buses, the number of connected bus. To implement ADMM, agents also have channels to exchange information with neighbour agents to update data in each iteration. The agent A is a neighbour of agent B if the bus corresponding to agent A and the bus corresponding to agent B are connected by a power line. Therefore, the number of agents is equal to the number of buses and the communication topology of multi-agent system depends on the connection graph of grid network.

From the general form of consensus problem and optimal power flow problem, the variables of the sub-problem in an agent are voltage vector of local and neighbour buses. In ADMM method, these variables are local copies of global variables of bus voltages. Figure 1 shows the construction of variables in agent $i$. In this example, agent at node $i$ has capability of connecting to agents at node $j$ and node $k$. Each edge represents a consensus constraint between a local variable component and a global variable.

Agents were designed to implement the iterative ADMM method. The operation of agent $i$ at iteration $l$ is summary as following:

*Step 1)* Finding solutions $\hat{V}_l^i$ of its augmented local objective function subjected to local constraints

- If $i$ is the generator node:

$$\underset{\hat{V}^i}{\text{minimize}} \quad \frac{1}{2}\hat{V}^{iT}\cdot\mathbf{Z}_{\mathbf{P}}^i\cdot\hat{V}^i + y^{iT}\hat{V}^i + \frac{\rho}{2}\left\|\hat{V}^i - Z^i\right\|_2^2$$

Subject to

$$P_{\min}^i \le \hat{V}^{iT}\cdot\mathbf{Z}_{\mathbf{P}}^i\cdot\hat{V}^i \le P_{\max}^i$$

$$Q_{\min}^i \le \hat{V}^{iT}\cdot\mathbf{Z}_{\mathbf{Q}}^i\cdot\hat{V}^i \le Q_{\max}^k$$

$$V_{\min}^2 \le (V^{i\,\mathrm{Re}})^2 + (V^{i\,\mathrm{Im}})^2 \le V_{\max}^2$$

- If $i$ is the load node

$$\underset{\hat{V}^i}{\text{minimize}} \quad \frac{1}{2}\hat{V}^{iT}\cdot\mathbf{Z}_{\mathbf{P}}^i\cdot\hat{V}^i + y^{iT}\hat{V}^k + \frac{\rho}{2}\left\|\hat{V}^i - Z^i\right\|_2^2$$

Subject to

$$\hat{V}^{iT}\cdot\mathbf{Z}_{\mathbf{P}}^i\cdot\hat{V}^i - P_{load}^i = 0$$

$$\hat{V}^{iT}\cdot\mathbf{Z}_{\mathbf{Q}}^i\cdot\hat{V}^i - Q_{load}^i = 0$$

$$V_{\min}^2 \le (V^{i\,\mathrm{Re}})^2 + (V^{i\,\mathrm{Im}})^2 \le V_{\max}^2$$

*Step 2)* Exchanging information with agent $j$ and agent $k$ to update the estimation of global variable $Z_l^k$

$$Z_l^i = \frac{1}{k_g}\sum_{m=i,j,k}\left(\left(\hat{V}_l^i\right)_m + \frac{\left(y_{l-1}^i\right)_m}{\left(\rho^i\right)_m}\right)$$

where $k_g$ is the number of neighbour and itself, $k_g = 3$ in this case.

*Step 3)* Updating the local Lagrange multipliers

$$y_l^i = y_{l-1}^i + \rho\left(\hat{V}_l^i - Z_l^i\right)$$

*Step 4)* Moving to the next iteration

$$l = l + 1$$

The number of iteration for reaching the consensus value depends on the scale of system and communication topology.

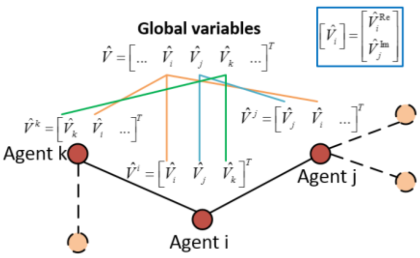


Figure 1. Local and global voltage vector

## Agent structure

In this research, we developed the agents in *python*, a high level and scientific program language. Agents are applications designed to compute and communicate. They can be connected to hardware devices or real-time power simulators and update measurement signals of grid simulation. Agents talk with neighbours by using *gRPC*, a high performance, open-source RPC (Remote procedure call) framework. An agent consists of an *gRPC* server and *gRPC* clients of servers hosted in neighbours. The structure of agents is illustrated in Figure 2. This is what happen in agents in one iteration: agent seeks the optimal point of the local objective, share and collect information with neighbors and then move to next one. The multi-agent system reaches the equilibrium point of OPF problem after a number of iteration.

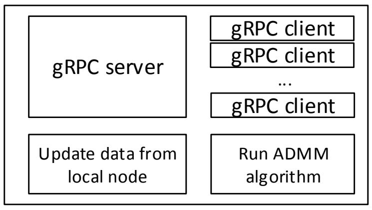


Figure 2. The structure of a single agent

## CASE STUDY

In order to validate the proposed methodology, we used a 6-bus grid including 3 generators and 3 loads with parameters in Table 1. We chose a meshed topology system which is more complicated than radical topology to prove the precise of the method in a more general case. The agent based distributed OPF was deployed to minimize active power loss of the grid. Figure 3 illustrates the test case grid and the inter-agent communication network. The multi-agent system was created by 6 applications running simultaneously in distinguish processes corresponding to 6 agents located at 6 buses of system. The data transmission is implemented through different ports under *gRPC* protocol. A range of penalty parameter $\rho$ is alternately applied to consider the convergent performance. The convergence of optimal DERs active and reactive power output of generators is shown in Figure 4 and Figure 5. In all cases, the output values were reached the stable state but after different iterations. The results also shown the effect of $\rho$ into the operation of agents and the important of choosing $\rho$.

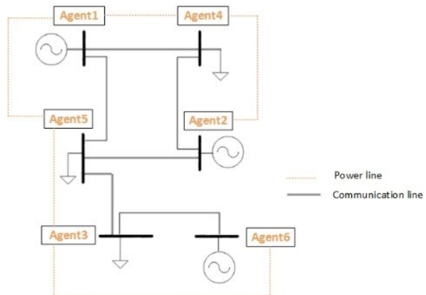


Figure 3. Testing microgrid with multi-agent system

The system can properly achieve consensus with the best performance ADMM at $\rho$ =25. At this value of $\rho$, the optimal values were found after ~300 iterations. From the output of agents, the total active power loss is $\sum \Delta P_{loss} = 0.0874\,pu$. In comparing with centralized method using *pypower* $\sum \Delta P_{loss}(centralized) = 0.0865 pu$, the result of ADMM distributed method is slightly higher but almost identical and can be considered to improve in next works.

Table 1. Parameter of testing grid

| Bus | P load (pu) | Q load (pu) | Line | Impedance (pu) |
|---|---|---|---|---|
| 3 | 0.7 | 0.7 | 1-4 | 0.05+j0.2 |
| | | | 1-5 | 0.08+j0.3 |
| 4 | 0.7 | 0.7 | 2-4 | 0.05+j0.1 |
| | | | 2-5 | 0.1+j0.3 |
| 5 | 0.7 | 0.7 | 3-5 | 0.12+j0.26 |
| | | | 3-6 | 0.02+j0.1 |

## CONCLUSIONS

In this paper, the optimal power flow problem was formulated and decomposed into sub-problems for distributed solving approach. Agent with capability of computation, communication and online operation was designed for deploying ADMM method. A test case system was also implemented to show the proper of the method and the work of agent system.

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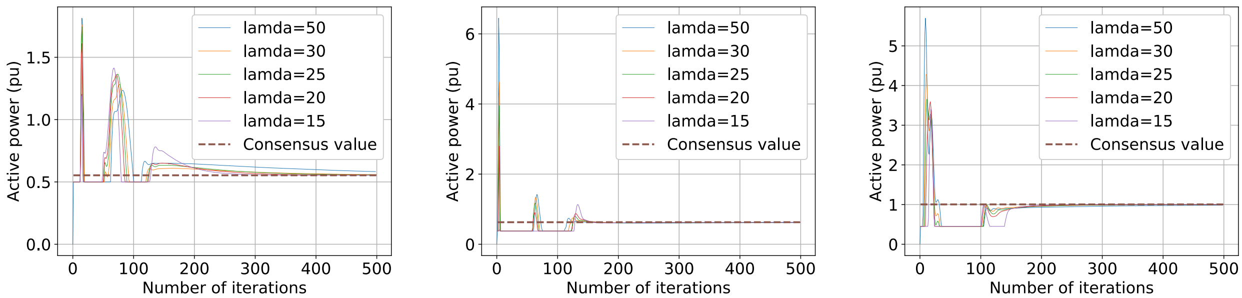
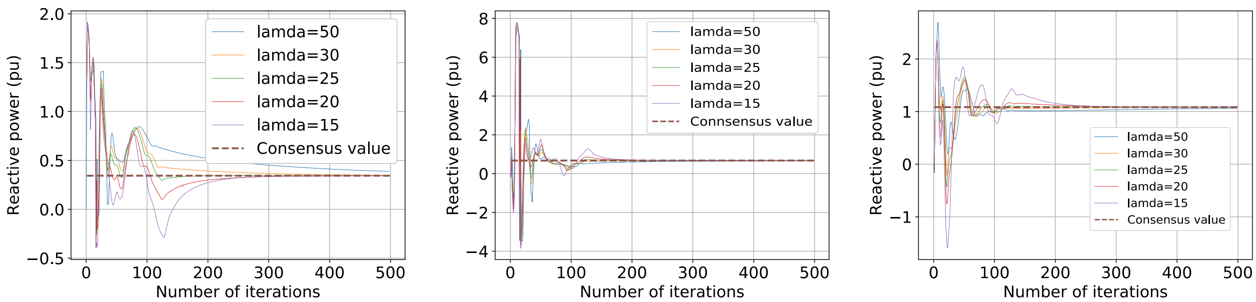



a) Generator 1 b) Generator 2 c) Generator 6

Figure 4. Convergence of active power output of generators for different values of $\rho$




a) Generator 1 b) Generator 2 c) Generator 6

Figure 5. Convergence of reactive power output of generators for different values of $\rho$